Kuwait University — Math 102 — March 26
Dept. of Math. and Comp. Sc. — First Examination — Duration: 75 min

Calculators and mobile phones are not allowed.

Answer all of the following questions.

Each (sub)question is worth 4 points.

Find $\frac{dy}{dx}$ if

(a) $$y = \frac{(\cos x)^{(\tan 2x)}\sqrt{x^5 + 6x}}{\ln(\sec x)},$$

(b) $$x\sin^{-1} y + xe^{\cosh y} - xy = x + y.$$

2. Show that

$$f(x) = \sqrt{3 + \ln(x^2 + e)} \qquad (x \le 0)$$

is a one-to-one function. Find $f^{-1}(x)$, state the domain and range of f^{-1}.

3. Show that the function $f(x) = (\tan^{-1} x - x)(x \in \mathbb{R})$ is decreasing. Find an equation of the tangent line to the graph of f^{-1} at the point $P\left(\frac{\pi - 4}{4}, 0\right)$.

4. Prove that for all $p > 0, q > 0$

$$\ln(pq) = \ln p + \ln q.$$

5. Write $\cos\left(\sec^{-1} x + \frac{\pi}{6}\right)$ as an algebraic expression in x if $x \ge 1$.

6. Prove that for all real numbers x

$$\operatorname{sech}^2 x = \frac{2\operatorname{sech} 2x}{1 + \operatorname{sech} 2x}$$

7. Evaluate the following integrals

(a) $\int \frac{\cosh x\, dx}{\sqrt{1 - \sinh^2 x}}$ (b) $\int \frac{dx}{\sqrt{4x - 4}}$, (c) $\int \frac{dx}{x(2 - \ln x^2)}$